An Information-Theoretic Approach to Model Identification in Interactive Influence Diagrams

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Outline

▶ Problem Statement
▶ Related Work
▶ Interactive Influence Diagrams (I - ID)
▶ Bayesian Model Identification
▶ Information-Theoretic Model Identification
▶ Experimental Results
Guess Your Opponent!

- Repeated Games
  - Observe previous actions
  - Predict next actions
  - Win the rewards

- Model Opponent
  - How and What will he/she play?
Related Work

Review

- Carmel & Markovitch (1996)
  - Model agents’ strategies using finite state automata
- Suryadi & Gmytrasiewicz (1999)
  - Learn influence diagrams to be consistent with observations
- Saha et al. (2005)
  - Approximate agents’ decision functions using Chebyshev polynomials
Interactive Influence Diagram (I - ID, Doshi et al. 2007)

- A generic level $l$ Interactive-ID (I-ID) for agent $i$ situated with one other agent $j$
  - **Model Node:** $M_{j,l-1}$
  - Models of agent $j$ at level $l - 1$
  - **Policy link:** dashed line
  - Distribution over agent $j$’s actions given its models
  - **Beliefs on** $M_{j,l-1}$: $P(M_{j,l-1} | s)$
  - Be updated over time
Details of the Model Node

- Members of the model node
  - Different chance nodes: solutions of models $m_{j,l-1}$
  - $Mod[M_j]$ represents the different models of agent $j$
- CPT of the chance node $A_j$ is a multiplexer
  - Assumes the distribution of each of the action nodes ($A_j^1$, $A_j^2$) depending on the value of $Mod[M_j]$
Public Good (PG) Game

- There are two agents initially endowed with $X_T$ amount of resources. Each agent may choose: Fully Contribute ($FC$), Partially Contribute ($PC$) the resources to a public pot, or not contribute ($D$: called defect here)
Public Good (PG) Game

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- The value of resources in the public pot is discounted by $c_i$ ($\leq 1$) for each agent $i$, where $c_i$ is the marginal private return
Public Good (PG) Game

- There are two agents initially endowed with \( X_T \) amount of resources. Each agent may choose: Fully Contribute (FC), Partially Contribute (PC) the resources to a public pot, or not contribute (D: called defect here)
- The value of resources in the public pot is discounted by \( c_i \) (\( \leq 1 \)) for each agent \( i \), where \( c_i \) is the marginal private return
- In order to encourage contributions, the contributing agents punish free riders \( P \) but incur a small cost \( c_p \) for administering the punishment
Payoff Matrix

<table>
<thead>
<tr>
<th></th>
<th>FC</th>
<th>PC</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC</td>
<td>$2c_iX_T$, $2c_jX_T$</td>
<td>$\frac{3}{2}X_Tc_i - \frac{1}{2}c_p$, $\frac{1}{2}X_T + \frac{3}{2}X_Tc_j - \frac{1}{2}P$</td>
<td>$c_iX_T - c_p$, $X_T + c_jX_T - P$</td>
</tr>
<tr>
<td>PC</td>
<td>$\frac{1}{2}X_T + \frac{3}{2}X_Tc_i - \frac{1}{2}P$, $\frac{3}{2}X_Tc_j - \frac{1}{2}c_p$</td>
<td>$\frac{1}{2}X_T + c_iX_T$, $\frac{1}{2}X_T + c_jX_T$</td>
<td>$\frac{1}{2}X_T + \frac{1}{2}c_iX_T - \frac{1}{2}P$, $X_T + \frac{1}{2}c_jX_T - P$</td>
</tr>
<tr>
<td>D</td>
<td>$X_T + c_iX_T - P$, $c_jX_T - c_p$</td>
<td>$X_T + \frac{1}{2}c_iX_T - P$, $\frac{1}{2}X_T + \frac{1}{2}c_jX_T - \frac{1}{2}P$</td>
<td>$X_T$, $X_T$</td>
</tr>
</tbody>
</table>

Table: PG game with punishment. Based on punishment, $P$, and marginal return, $c_i$, agents may choose to contribute than defect.
Agent $j$’s Types

- $m_j^1$: A reciprocal agent who contributes only when it expects the other agent to contribute as well
  - Low values of $c_i$
- $m_j^2$: An altruistic agent who prefers to contribute during the play
  - High values of $c_i$
- $m_j^3$: Relies on both its own and opponent actions in the previous time step
- $m_j^4$: Relies more on the past interaction - up to two previous time steps
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Our Representation

I-ID for PG Game
Two Cases

- Case 1: $m_j^* \in M_j$ (Traditional)
  - Bayesian Model Identification
- Case 2: $m_j^* \notin M_j$
  - Information-Theoretic Model Identification
If an agent's prior belief assigns a non-zero probability to the true model of the other agent, its posterior beliefs updated using Bayesian learning will converge with probability 1.

Don’t always converge to the true model of the other agent.

Observationally equivalent models.
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Case 1: $m_j^* \in M_j$ - Bayesian Model Identification

Observational Equivalence

Observational Equivalence

- Two $j$’s Models
  - **Model 1:** Select $FC$ for an infinite number of steps, but if at any time $i$ chooses $PC$, $j$ would also do so at the next time step and then continue selecting $PC$
  - **Model 2:** Play tit-for-tat strategy: $j$ performs the action which $i$ did in the previous time step
  - $i$ selects $FC$ for an infinite number of times
Case 2: \( m_j^* \not\in M_j \) - Information-Theoretic Model Identification

Relevant Models

- **Relevant model** \( m_j^n \)
  - A relevant model predicts an action that is likely to correlate with a particular observed action of the other agent
    - \( Pr(a_j^1 | m_j^n, a_j^*) \geq Pr(a_j^1 | m_j^n, \bar{a}_j^*) \), where \( a_j^1 \in OPT(m_j^n) \)
  - We interpret the existence of a mutual pattern as evidence that the candidate model shares some behavioral aspects of the true model

- Assign large probabilities to \( m_j^n \) in \( Mod[M_j] \) over time
Learning Naive Bayesian Models

Figure: History of interaction
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Case 2: \( m_j^* \not\in M_j \) - Information-Theoretic Model Identification

Mutual Information

Mutual Information as Model Weight

\[
MI(m^n_j, m^*_j) \overset{\text{def}}{=} Pr(A^n_j, A_j) \log \left[ \frac{Pr(A^n_j, A_j)}{Pr(A^n_j) Pr(A_j)} \right] \\
= Pr(A^n_j | A_j) Pr(A_j) \log \left[ \frac{Pr(A^n_j | A_j)}{Pr(A^n_j)} \right]
\]  \hspace{1cm} (2)

- \( A^n_j \): the chance node mapped from \( m^n_j \)
- \( A_j \): the observed actions generated by \( m^*_j \)
Case 2: $m^*_j \not\in M_j$ - Information-Theoretic Model Identification

Algorithm

Model Weight Update

**Step 1:** Update the training set using $i$’s observations and model $m^+_j$ solutions
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Case 2: \( m^*_j \notin M_j \) - Information-Theoretic Model Identification

Algorithm

Model Weight Update

Step 1: Update the training set using \( i \)'s observations and model \( m^j_p \) solutions

Step 2: Learn the parameters of the naive BN including the chance nodes \( A^1_j, \ldots, A^n_j \), and \( A_j \)
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Case 2: $m^*_j \not\in M_j$ - Information-Theoretic Model Identification

Algorithm

Model Weight Update

**Step 1:** Update the training set using $i$’s observations and model $m^p_j$ solutions

**Step 2:** Learn the parameters of the *naive BN* including the chance nodes $A^1_j, \ldots, A^n_j$, and $A_j$

Loop
Case 2: $m^*_j \not\in M_j$ - Information-Theoretic Model Identification

Algorithm

Model Weight Update

Step 1: Update the training set using $i$’s observations and model $m^p_j$ solutions

Step 2: Learn the parameters of the *naive BN* including the chance nodes $A^1_j, \ldots, A^n_j$, and $A_j$

Loop

Step 3: Compute $MI(m^p_j, m^*_j)$
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Case 2: $m_j^* \not\in M_j$ - Information-Theoretic Model Identification

Algorithm

Model Weight Update

Step 1: Update the training set using $i$’s observations and model $m_j^p$ solutions

Step 2: Learn the parameters of the naive BN including the chance nodes $A_j^1, \ldots, A_j^n$, and $A_j$

Loop

Step 3: Compute $MI(m_j^p, m_j^*)$

Step 4: Obtain $Pr(A_j|A_j^p)$ from the learned naive BN
Model Weight Update

Step 1: Update the training set using i’s observations and model $m^p_j$ solutions
Step 2: Learn the parameters of the naive BN including the chance nodes $A^1_j, \ldots, A^n_j$, and $A_j$
Loop
   Step 3: Compute $MI(m^p_j, m^*_j)$
   Step 4: Obtain $Pr(A_j|A^p_j)$ from the learned naive BN
   Step 5: Populate CPD row of the chance node $A_j$ using $Pr(A_j|A^p_j, m^p_j)$
Model Weight Update

Step 1: Update the training set using i’s observations and model $m_j^p$ solutions

Step 2: Learn the parameters of the naive BN including the chance nodes $A_{j1}, \ldots, A_{jn}$, and $A_j$

Loop

Step 3: Compute $MI(m_j^p, m_j^*)$

Step 4: Obtain $Pr(A_j|A_{j1}^p)$ from the learned naive BN

Step 5: Populate CPD row of the chance node $A_j$ using $Pr(A_j|A_{j1}^p, m_j^p)$

Step 6: Normalize $MI(m_j^p, m_j^*)$
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Case 2: \( m^*_j \not\in M_j \) - Information-Theoretic Model Identification

Algorithm

Model Weight Update

**Step 1:** Update the training set using \( i \)'s observations and model \( m^p_j \) solutions

**Step 2:** Learn the parameters of the *naive BN* including the chance nodes \( A^1_j, \ldots, A^n_j \), and \( A_j \)

Loop

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**Step 4:** Obtain \( Pr(A_j|A^p_j) \) from the learned *naive BN*

**Step 5:** Populate CPD row of the chance node \( A_j \) using \( Pr(A_j|A^p_j, m^p_j) \)

**Step 6:** Normalize \( MI(m^p_j, m^*_j) \)

**Step 7:** Populate CPD of the chance node \( Mod[M_j] \) using \( MI \)
Some Properties

► Property 1
  ▶ Irrelevance: $Pr(a_j|m^n_j, a^*_j) = Pr(a_j|m^n_j, \bar{a}^*_j)$
  ▶ $MI(m^n_j, m^*_j) = 0$
Some Properties

- **Property 1**
  - Irrelevance: $\Pr(a_j|m_j^n, a^*) = \Pr(a_j|m_j^n, \bar{a}^*)$
  - $MI(m_j^n, m^*) = 0$

- **Property 2**
  - Relevance Ordering ($m_j^n$ is more relevant than $m_j^p$):
    
    $Pr(a_j^1|m_j^n, a^*) \geq Pr(a_j^1|m_j^p, a^*)$ and
    
    $Pr(a_j^1|m_j^n, \bar{a}^*) \leq Pr(a_j^1|m_j^p, \bar{a}^*)$
  
  - Larger $MI$ is assigned to $m_j^n$: $MI(m_j^n, m^*) \geq MI(m_j^p, m^*)$
Some Properties

- **Property 1**
  - Irrelevance: \( Pr(a_j|m_j^n, a_j^*) = Pr(a_j|m_j^n, \bar{a}_j^*) \)
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    Pr(a_j^1|m_j^n, \bar{a}_j^*) \leq Pr(a_j^1|m_j^p, \bar{a}_j^*)
    \]
  - Larger \( MI \) is assigned to \( m_j^n \): \( MI(m_j^n, m_j^*) \geq MI(m_j^p, m_j^*) \)

- **Property 3**
  - Convergence
  - Given that the true model \( m_j^* \in M_j \) and is assigned a non-zero probability, the normalized distribution of mutual information of the models converges with probability 1
MI Equivalence

- One example
  - True model: $j$ always plays $FC$
  - Candidate model: $j$ always plays $D$
  - Both models are assigned equal MI
    - Dependency is elicited between $D$ and $FC$

- Set of MI equivalence $\supseteq$ Set of Observational equivalence

- NOT affect prediction performance
  - The perceived dependency classifies $D$ into $FC$ through the learned parameters $Pr(A_j|A_j^p)$
Method Evaluation

- Methods
  - Bayesian Learning ($BL$)
  - Mutual Information ($MI$)
  - Adaptation Bayesian Learning ($A-BL$)
    - Restart the BL process when the likelihoods become zero by assigning candidate models prior weights using the frequency with which the observed action has been predicted by the candidate models so far
  - KL Divergence
    - Measure difference between $A^n_j$ and $A_j$ distributions
Method Evaluation

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  - Measure difference between \(A_j^n\) and \(A_j\) distributions

Scenarios

- PG Games
- Negotiation Games (4 types of opponents)
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Experimental Results

Case 1: \( m_j^* = m_j^4, M_j = \{ m_j^1, m_j^3, m_j^4 \} \)

Case 2: \( m_j^* = m_j^1, M_j = \{ m_j^2, m_j^3, m_j^4 \} \)
Conclusions

- I-ID in Repeated Games
- Two Cases for Model Identification in I-ID
- MI Complements BL